Calculus I [MATH161-1]

Final Exam (Spring, 2022)

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1. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \sin\left(\frac{\cos(n\pi)}{n}\right)$

(b) $\sum_{n=3}^{\infty} \frac{\sin\left(\tan^{-1}\frac{1}{3n}\right)}{\ln n}$

2. Find the interval of convergence of the series.

$\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!} x^n$

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3. (a) Use the definitions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

and the Maclaurin series for e^x to find the Maclaurin series for $f(x) = \sinh x \cosh x$.

(b) Use the Maclaurin series for $\ln(1+x)$ to find the Maclaurin series for $g(x) = \ln\left(\frac{1+x}{1-x}\right)$, where $|x| < 1$.

(c) Use (a) and (b) to find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{3^{n+1}}{(2n)!} + \frac{1}{3^n}\right)$$

4. Find the values of p for which the series is convergent. $\sum_{n=2}^{\infty} \frac{n + \sqrt{\ln(n^n)}}{1 + n^p}$

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5. (a) Find the Taylor polynomial $T_{15}(x)$ for $\tan(x^3)$ centered at $a=0$.

(b) Use (a) and Maclaurin series for functions to find the limit. (Do **NOT** use L'Hospital's rule to find the limit.)

$$\lim_{x\to 0}\frac{2\sin(x^3)+\tan(x^3)-3x^3}{x^5-\tan^{-1}(x^5)}$$

6. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be vectors with

$$|\mathbf{v}_1|=3,\quad |\mathbf{v}_2|=5,\quad \text{and}\quad \mathbf{v}_1\cdot\mathbf{v}_2=9.$$

If $\mathbf{v}_n=\mathrm{proj}_{\mathbf{v}_{n-2}}\mathbf{v}_{n-1}$ for $n\ge 3$, evaluate $\sum_{n=1}^{\infty}|\mathbf{v}_2\times\mathbf{v}_{2n+1}|$.

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7. Let **a**, **b**, **c**, **f**, **g** and **h** be non-zero vectors.

(a) Find the scalar projection of **a** onto **c**
if $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$, $4\,\mathrm{proj}_{\mathbf{c}}\mathbf{a}=\mathrm{proj}_{\mathbf{c}}\mathbf{b}$, and $|\mathbf{c}|=2$.

(b) Find $(\mathbf{g}+3\mathbf{h})\cdot(\mathbf{f}\times 2\mathbf{g})+2\mathbf{g}\cdot(\mathbf{f}\times 4\mathbf{h})$
if $\mathbf{f}\cdot(\mathbf{g}\times\mathbf{h})=-2$.

8. (a) Find symmetric equations for the tangent line L to the curve of intersection of the parabolic cylinder $y=x^2$ and the paraboloid $z=4y^2+x^2$ at the point $(1,1,5)$.

(b) Let Φ be the plane that passes through the point $(3,-1,3)$ and contains the line

$$x=2t-1,\quad y=3t+1,\quad z=1-t.$$

Find an equation of the plane that contains the line L of (a) and is perpendicular to the plane Φ.

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9. Let C: $\mathbf{r}(t) = \langle \sqrt{2}\, e^t,\ e^t \sin t,\ e^t \cos t \rangle$.

(a) Find the arc length function for the curve C measured from $P(\sqrt{2}, 0, 1)$ in the direction of increasing t.

(b) Reparametrize the curve C with respect to arc length measured from $P(\sqrt{2}, 0, 1)$ in the direction of increasing t.

(c) Find the point Q on the curve C if the length of the curve from $P(\sqrt{2}, 0, 1)$ to Q is 10.

10. Let C: $\mathbf{r}(t) = \left\langle 1-t,\ \frac{1}{3}t^3 - t + \frac{2}{3},\ 0 \right\rangle$ and let P be the point on C where the tangent line of C is parallel but not equal to the x-axis.

(a) Find the point P and the normal plane of C at P.

(b) Find the binormal vector of C at P.

(c) Find an equation of the osculating circle of C at P by regarding C as a curve in the xy-plane.