Calculus I (MATH 161) Final Exam (Spring, 2022)

Department : Id number : Name :

풀이 과정을 자세히 기술해야 합니다.

1. Test the series for convergence or divergence, give reasons for your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!e^{\sqrt{n}}}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2 + \ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\left(1-\cos\left(\frac{1}{n}\right)\right)$

2.

(a) Find the Maclaurin series for xe^x.

(b) Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{n2^n}{(n+1)!}$$

Department : Id number : Name :

3. Let $f(x) = \arcsin(x)$.

(a) Find $f^{(57)}(0)$.

(b) Estimate $\arcsin(-0.1)$ with |error| $< 10^{-2}$.

4. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in V_3 satisfying

$$|\mathbf{u} + \mathbf{v}| = 5, \ |\mathbf{u} - \mathbf{v}| = 3$$

(a) Evaluate $|\mathbf{u}|^2 + |\mathbf{v}|^2$.

(b) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

5. Let α be a plane that is parallel to the plane $x-y-z-1=0$ and contains the line $\frac{x}{3}=y+4=\frac{z}{2}$.
Let β be a plane that passes through the point $(2,1,1)$ and contains the line of intersection of the planes $x+2y-z=3$ and $2x-y+3z=-4$.

Find the angle between the planes α and β.

6. Consider points $P(1,0,0)$, $Q(0,1,0)$, and $R(0,0,1)$. Let W be any point in the line L passing through the points $(2,0,0)$ and $(0,1,1)$.

(a) Explain geometrically why the volume of the parallelepiped with adjacent edges PQ, PR, and PW is constant.

(b) Evaluate the volume.

Department : Id number : Name :

7. Find the osculating circle of $xy=2$ at the point $(1,2)$.

8. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=3}^{\infty} \frac{x^n}{(\ln n)^{\ln n}}$$

Department : Id number : Name :

9. (a) Find a vector function $\mathbf{r}(t)$ that parameterize the curve of intersection of the cylinder $x^2+z^2=4$ and the plane $y+z=2$.

(b) Find equation of the normal plane P_1 of the curve $\mathbf{r}(t)$ at the point $(\sqrt{3},1,1)$.

(c) Find equation of the plane P_2 that is perpendicular to the plane P_1 and contains the line

$$\mathbf{u}(t)=(2t-1)\mathbf{i}+(t+3)\mathbf{j}+(2-t)\mathbf{k}$$

10. Find all p such that the series is convergent.

$$\sum_{n=3}^{\infty}(\ln n)^p \tan\left(\frac{1}{n}\right)$$