Calculus I (MATH 161)

Midterm Exam (Spring, 2022)

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1. (15 pts.) Evaluate the limit, if it exists. If not, give reasons.

(a) $\lim_{x\to 0^+} \dfrac{x^2-1+\sin(\cos^{-1}x)}{x^3}$.

(b) $\lim_{x\to 0} \tanh\left(\dfrac{1}{x^2}\right)(2x+1)^{\cot x}$.

2. (15 pts.)

(a) Find the equation of the tangent line to the curve

$$\tan^{-1}(x^2y+y)=\frac{1}{2}\cos^{-1}(x+xy^3) \quad \text{at } x=0.$$

(b) Find the linearization of the function

$$f(x)=\cos^{-1}(\tanh x)$$

at $x=0$.

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3. (15 pts.) Show that

$$|\ln x - \ln y| \geq |x - y|$$

for all $x, y \in (0,1)$.

4. (15 pts.)

(a) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ for $x \in \mathbb{R}$ and $\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$ for $x \geq 1$.

(b) Find the volume of the solid obtained by rotating the region bounded by $y = \ln(x + \sqrt{x^2+1})$, $y = \ln\left(x + \frac{1}{2} + \sqrt{\left(x+\frac{1}{2}\right)^2 - 1}\right)$ and $y = 0$ about the y−axis.

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5. (15 pts.) Evaluate the following integral

$$\int \frac{x^4-x^3+4x^2+x+1}{x(x^2+1)^2}dx.$$

6. (15 pts.) Find the values of p for which the integral converges.

$$\int_2^\infty \frac{1}{(x^5-x)^p}\,dx.$$

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7. (15 pts.) Let $f(b)$ be the area of the surface generated by rotating the curve $y=\sinh^{-1}x, 0\le x\le b$, about the x-axis and $g(b)$ be the area of the surface generated by rotating the curve $y=\sinh^{-1}x, 0\le x\le b$, about the y-axis.

Compute $$\lim_{b\to 0^+}\frac{f(x)}{g(x)}.$$

8. (15 pts.) Find the arc length of the curve C from $t=0$ to the first point where there is a horizontal tangent line.

$$C:\ x(t)=2\cos t-\cos 2t\ ,\quad y(t)=2\sin t-\sin 2t.$$

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9. (15 pts.) Consider the region Ω in the first quadrant between two polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$. Find the volume of the solid generated by revolving the region Ω about the y-axis.

10. (15 pts.)

(a) Graph the curve $r = 2 - \cos(\frac{\theta}{2})$.

(b) Find the area of the region that lies inside both curves $r^2 = \sqrt{2}\sin 4\theta$ and $r = 1$.