Calculus I [MATH161-1]

Midterm Exam (Spring, 2022)

Department : Id number : Name :

1.

(a) Find an equation of the tangent line to the curve

$$\sin^{-1}(2-xy^2)+1=y^x$$

at the point $(2, 1)$.

(b) Find an equation of the normal line to the polar curve

$$r=2+\sin 3\theta$$

at the point specified by $\theta=\pi/3$

2. Suppose that C is the curve defined by the parametric curve $x=\sin 3t,\ y=\cos 2t,\ 0\le t\le 2\pi$.

(a) Find the points on C where the tangent is horizontal or vertical.

(b) Find $\dfrac{d^2y}{dx^2}$.

Department : Id number : Name :

3. Evaluate $\lim_{x\to\frac{\pi}{2}^-}\left(\frac{1-k\cot x}{1+k\cot x}\right)^{\sec x}$, where k is a non zero constant.

4. Suppose that $f(x)$ is differentiable and $f'(x)$ is continuous with $f(x)\geq 0$, $f(3)=8$, and $f'(3)=\sqrt{3}$.

(a) Let $s(x)$ be an arc length for a curve $y=f(x)$ from the starting point $P_0(0, f(0))$ to the point $P(x, f(x))$ and $s(3)=10$.
Use a linear approximation to estimate $s(3.01)$.

(b) Let $g(t)$ and $h(t)$ be the area of the surface generated by rotating the curve $y=f(x)$, $0\leq x\leq t$, about the x-axis and y-axis, respectively.
Compute differentials dg and dh for $t=3$ and $dt=\triangle t=0.01$.

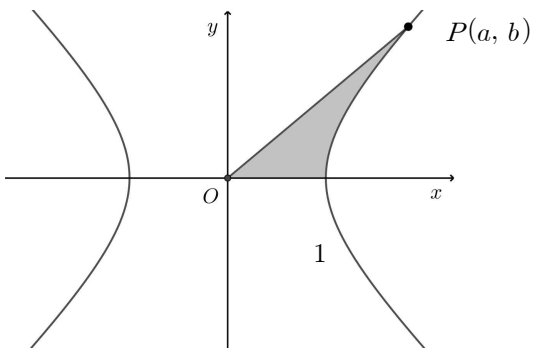
Department : Id number : Name :

5.

(a) Use differentiation to show that

$$\cosh^{-1} x = x\sqrt{x^2-1} - 2\int_1^x \sqrt{t^2-1}\,dt.$$

(b) The area of the shaded hyperbolic sector in the following figure is 2.
Use (a) to find the point P in the first quadrant.



$x^2 - y^2 = 1$

6. Evaluate the integrals

(a) $\int \sin(2\tan^{-1}x)dx$

(b) $\int \sinh x \tan^{-1}(\sinh x)dx$

Department : Id number : Name :

7. $\int_1^{\infty} \frac{5x^2+6x-5}{x^p(x+1)(x^2+4x+5)}\,dx$

(a) When $p=1$, evaluate this integral.

(b) Find the values of p for which the integral converges.

8. The integral represents the volume of the solid obtained by rotating the region R about the y-axis.

$$\int_0^{1/\sqrt{3}} \pi(\sinh^{-1}y)^2\,dy + \int_{1/\sqrt{3}}^{1} \pi(\ln y)^2\,dy$$

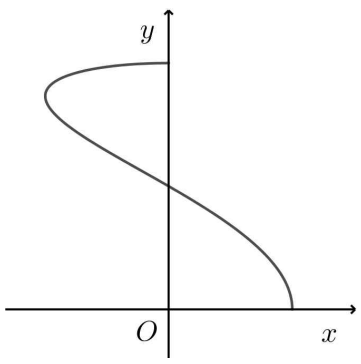
(a) Sketch the region R.

(b) Use the method of cylindrical shells to find the volume of the solid.

Department : Id number : Name :

9. Find the area of the region enclosed by the given parametric curve, $x-$axis and $y-$axis.

$$x=-\sin(3t),\ y=2\cos t,\ 0\le t\le\frac{\pi}{2}$$



10.

(a) Sketch the polar curves

$$r=2\sin\theta+2\cos\theta,\quad r^2=12\sin2\theta$$

and find all points of intersection.

(b) Find the area of the region that lies inside both curves.