Calculus I [MATH 161]   Final Exam (Spring 2022)

Department :   Id number :   Name :

단답형: (1번~5번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

1.(6 pts.) Use the Maclaurin series for $f(x) = e^{-x^2}\sin x^2$ to find $f^{(10)}(0)$.

2.(6 pts.) Let $\boldsymbol{u} = \langle 3, 2, 1 \rangle$ and $\boldsymbol{v} = \langle 1, 0, 1 \rangle$. Then

(a) Find the orthogonal projection $\text{otho}_{\boldsymbol{v}}\boldsymbol{u}$ of $\boldsymbol{u}$ on $\boldsymbol{v}$.

(b) Find the area of the parallelogram determined by $\boldsymbol{u}$ and $\boldsymbol{v}$

3.(6 pts.) Find the distance between the skew lines

$$L_1 : \frac{x-3}{2} = \frac{y+1}{4} = 2 - z \text{ and}$$

$$L_2 : \frac{x-3}{2} = y - 2 = \frac{z+2}{2}.$$

4.(6 pts.) Let C be the curve of intersection of the surfaces $z = x^2 - y^2$ and $x^2 + y^2 = 4$. Find parametric equations for the tangent line to the curve C at the point $(\sqrt{2}, \sqrt{2}, 0)$.

5.(6 pts.) Find an equation of the tangent plane of the surface

$$x\tan^{-1}(y^2) + 2xyz^2 = zx - y + 1$$

at the point $(1, 0, -1)$.

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Department : Id number : Name :

단답형: (6번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

6.(6 pts.) Let $w = xy + yz$, $x = r\sin\theta$, $y = r^2(\sin\theta)^2$, and $z = r^2\sin\theta$. Find $\frac{\partial w}{\partial\theta}$ at $(r,\theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$.

7.(6 pts.) Let $f(x,y) = \int_{xy}^{x^2+y^2} e^{-2(t-1)^2} dt$. Evaluate $f_x(1, 0)$.

8.(6 pts.) Find the directional derivative of the function $f(x,y,z) = \frac{x^{2yz}}{2}$ at the point $(e, 2, 1)$ in the direction of the vector $\boldsymbol{v} = 3\mathbf{j} + 4\mathbf{k}$.

9.(6 pts.) Find (a), (b), and (c) such that

$$\iint_R (2-x)\,dA ,$$
$$= \int_{\boxed{(a)}}^{0} \int_{-x-1}^{x+1} (2-x)\,dy\,dx + \int_0^1 \int_{\boxed{(b)}}^{\boxed{(c)}} (2-x)\,dy\,dx,$$

where $R = \{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \le 1\}$.

10.(6 pts.) Find the volume of the solid enclosed by the surface $z = x\sec^2 y$ and the planes

$x = 0$, $x = 2$, $y = 0$, $y = \frac{\pi}{4}$, $z = 0$.

Department : Id number : Name :

<u>**서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.**</u>

11.(15 pts.) Find a power series representation for the function $f(x) = x^2 \tan^{-1}(4x^2)$ and determine the radius of convergence.

12.(15 pts.) The surface $S: z = x^2 + y^2 - 1$ and the curve $C: \boldsymbol{r}(t) = \langle t,\ t^2,\ t^3 \rangle\ (t \geq 0)$ intersect at a point P in the first octant. Find the angle of intersection between the surface S and the curve C at P. (This is the angle between the tangent plane to the surface and the tangent vector to the curve.)

Department : Id number : Name :

13.(15 pts.) Let $\overline{D}_{\boldsymbol{u}} f(x,y,z)$ be the directional derivative of the function f at (x,y,z) in the direction of the vector $\boldsymbol{u}$. Let

$$f(x,\ y,\ z) = x + ay + bz + 7,$$

$\boldsymbol{u} = \langle 3,\ 0,\ -1 \rangle$, $\boldsymbol{v} = \langle 1, 1,\ -1 \rangle$, and $\boldsymbol{h} = \langle 1,\ -2, -2 \rangle$. If $\overline{D}_{\boldsymbol{v}} f(x,\ y,\ z) = \sqrt{3}$ and $\overline{D}_{\boldsymbol{h}} f(x,\ y,\ z) = 5$, then find $\overline{D}_{\boldsymbol{u}} f(x,\ y,\ z)$.

14.(15 pts.) Let

$$f(x,\ y, z) = 2(x - y + z)$$

and $D = \{(x,\ y, z) \mid x^2 + y^2 + z^2 - 4y \le 0,\ y \ge 2\}$.
Find the absolute maximum and minimum values of f on the set D.

Department : Id number : Name :

15.(20 pts.) Let $f(x,y)=12xy-2x^2y+4xy^2$.

(a) Find all critical points of $f(x,y)$.

(b) Find the local maximum and minimum values, and saddle points of $f(x,y)$.

(c) Find the absolute maximum and minimum values of $f(x,y)$ on the region T. Here T is the closed triangular region in the xy-plane with vertices $(0,0)$, $(6,0)$, and $(4,-4)$.

$$\int_{-\frac{\pi}{2}}^{0}\int_{-y}^{\frac{\pi}{2}}\sin(x^2)dxdy+\int_{0}^{\pi}\int_{\frac{1}{2}y}^{\frac{\pi}{2}}\sin(x^2)dxdy$$
$$=\int_{0}^{\frac{\pi}{2}}\int_{-x}^{2x}\sin(x^2)dydx=-\frac{3}{2}\left(\cos\left(\frac{\pi^2}{4}\right)-1\right)$$