# Calculus I [MATH 161]

**Midterm Exam (Spring 2022)**

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**1.**(20 pts.) Find the limit, if it exists. If the limit does not exist, explain why.

(a) $\lim_{x\to\infty}\frac{(x+1)\sin x^2}{x^2}$

(b) $\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{\sinh x}-\frac{1}{\tanh x}\right)$

(c) $\lim_{x\to 0^+}\left(1+\tan^{-1}x\right)^{-1/x}$

(sol)

**2.**(20 pts.) Evaluate the integral.

(a) $\int \tan^3 x \sec x \, dx$

(b) $\int e^x \sqrt{1+e^{2x}} \, dx$

(c) $\int_{\sqrt{3}}^{2}\frac{2x\left(x^2+11\right)}{\left(x^2-1\right)^2}\,dx$

(sol)

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**3**.(20 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2(-1)^n}{\sqrt[3]{(n+1)^2}}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{5^n n!}{n^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{3n^2}$

(sol)

**4**.(15 pts.) Let the curve $C$ be defined by

$$\ln(x+xy-y)+\tan^2\left(\cos^{-1}\left(\frac{1}{x\sqrt{y}}\right)\right)=1\,.$$

Then, find an equation of the tangent line to the curve $C$ at $x=1$.

(sol)

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**5.**(15 pts.) Evaluate the integral.

$$\int_0^2 \sqrt{\frac{2+x}{2-x}}\,dx$$

(sol)

**6.**(15 pts.) Let $I=\left[-\frac{2\pi}{3},\ \frac{4\pi}{3}\right]$. A curve $C$ is defined by the parametric equations

$$x=\cos t+t\sin t,\ y=\sin t-t\cos t \text{ for } t\in I.$$

(a) Find the points on the curve $C$ where the tangent is horizontal or vertical on $I$.

(b) Find $\frac{d^2y}{dx^2}$ for the curve $C$ when $t=\frac{\pi}{4}$.

(sol)

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**7**.(15 pts.) Answer the following questions.

(a) Graph the polar curves $r=2-2\sin\theta$ and $r=2\sin\theta$.

(b) Find the area of the region enclosed by the curves $r=2-2\sin\theta$ and $r=2\sin\theta$.

(sol)

**8**.(15 pts.) Let $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n5^n}$.

(a) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(b) Approximate the sum of the series correct to $|error| \le 0.0001$.
(**Do NOT** evaluate the sum of fractions.)

(sol)

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9.(15 pts.) Consider the power series

$$\sum_{n=2}^{\infty} \frac{(x+2)^n}{n\sqrt{\ln n}}.$$

(a) Find the radius of convergence of the power series.

(b) For what values of $x$ does the series converge?

sol)