Calculus II [MATH 162]

Final Exam (Fall 2022)

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1.(15pts.) Find the center of mass of the lamina bounded by the ellipse

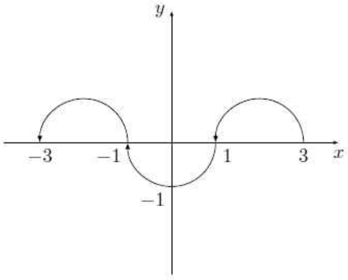
$$x^2 + 4(y-1)^2 = 4$$

if the density function is given by $\rho(x,y) = y$.

2.(10pts.) Let $\mathbf{F} = (x - \sin x)\mathbf{i} + (y^2 + e^y)\mathbf{j} + (x+z)\mathbf{k}$ and C be the curve given by
$\mathbf{r}(t) = (1 + \sin(20t))\cos t\,\mathbf{i} + (1 + \sin(20t))\sin t\,\mathbf{j} + \cos(20t)\mathbf{k}$
with $0 \le t \le 2\pi$.

Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3.(10pts.) Let $\mathrm{F}(x,y)=\dfrac{-y\mathrm{i}+x\mathrm{j}}{x^2+y^2}$. Compute the line integral $\int_C \mathrm{F}\cdot d\mathrm{r}$, where C is the union of three semicircles from (3,0) to (-3,0) as following.



4.(15pts.) Find the maximum value of $\iint_S \mathrm{F}\cdot d\mathrm{S}$ for any simple closed surface S oriented outward, where $\mathrm{F}=(x-x^3)\mathrm{i}+(y-y^3)\mathrm{j}+(z-z^3)\mathrm{k}$

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5.(15 pts.)

(1) Show that the vector field is conservative if

$$\mathbf{F}(x,y,z)=\left\langle \frac{e^z}{(1+x^2)^2},\ \sin^2 y,\ \frac{1}{2}\left(\frac{x}{x^2+1}+\tan^{-1}x\right)e^z \right\rangle$$

and find the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ if C is a curve from $(0,0,0)$ to $(1,\pi/2,1)$.

(2) Evaluate the line integral.

$$\int_C (xy^2+2z)\,ds$$

$C: x=\cos 2t, y=\sin 2t, z=t, 0\le t\le \frac{\pi}{4}$

6.(15 pts.)

Let $\mathbf{F}(x,y,z)=xz^2\,\mathbf{i}+(e^y-x^3)\mathbf{j}+(y^2-ze^y)\mathbf{k}$.

Find the flux of $\mathbf{F}$ across the part of the ellipsoid $x^2+y^2+4z^2=4$ that lies above the plane $z=0$ and is oriented upward.

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7.(20 pts.)

(1) Evaluate $\iint_S x^3 + z^2 \, dS$, where S is the surface $x^2 + y^2 + z^2 = 4$, $-1 \le z \le \sqrt{3}$.

(2)

Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 2y$.

8.(15pts.) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F} = (yz + 2xz)\mathbf{i} + (yz + \frac{z^2}{2})\mathbf{j} + (yx + yz)\mathbf{k}$$

and C is the boundary of the part of the ellipsoid

$$x^2 + \frac{y^2}{4} + z^2 = 1$$

in the first octant, oriented counterclockwise as viewed from above.

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9.(15 pts.) Verify that Stokes' Theorem is true for the given vector field

$$\mathbf{F}(x,y,z) = 2z\,\mathbf{i} + xy\mathbf{j} + (z+2y)\mathbf{k}$$

and the surface S which is the part of the plane $x-2z=3$ that lies inside of the cylinder $y^2+z^2=1$, oriented in the direction of positive x−axis.

10.

(1) 다음 연립방정식을 행렬을 이용하여 나타내고 크래머 공식을 이용하여 해 중 x_2를 구하라.(풀이 과정을 서술) (8점)

$$\begin{cases} x_1 + bx_2 + x_3 + x_4 = 1 \\ ax_1 + ax_4 = 0 \\ 2x_1 + x_2 + cx_3 + (2-c)x_4 = 1 \\ 3x_1 + x_3 + 2x_4 = 0 \end{cases}$$

(2) $A = \begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \\ x & y & z \end{pmatrix}$ 일 때 $|A| = k(\neq 0)$ 이다.

다음 주어진 행렬의 행렬식을 k 로 나타내어라.(단답형) (12점)

a. $B = 2\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \\ x & y & z \end{pmatrix}$, $\det(B) =$

b. $C = \begin{pmatrix} 6x & 3y & 3z \\ -4\alpha & -2\beta & -2\gamma \\ 2a & b & c \end{pmatrix}$, $\det(C) =$

c. $D = \begin{pmatrix} \alpha+x & a & 2x \\ \beta+y & b & 2y \\ \gamma+z & c & 2z \end{pmatrix}$, $\det(D) =$