Calculus II [MATH162]

Final Exam (Fall, 2022)

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1.(14 pts.) Use the change of variables to evaluate

$$\iint_R \sqrt{3}\cos(x^2 - xy + y^2)\, dA$$

where R is the region bounded by the ellipse $x^2 - xy + y^2 = 1$.

2.(14 pts.) Find the center of mass of a wire that lies along the curve $\mathbf{r}(t) = (t^2 - 1)\,\mathbf{j} + 2t\,\mathbf{k}$, $-1 \le t \le 1$, if the density is $\delta(x, y, z) = 15\sqrt{y+2}$.

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3.(15 pts.) Let $\mathbf{F}(x, y, z) = \dfrac{-y}{x^2+y^2}\,\mathbf{i} + \dfrac{x}{x^2+y^2}\,\mathbf{j} + e^z\,\mathbf{k}$ and C be a curve given by

$$\mathbf{r}(t) = \cos^2 t\,\mathbf{i} + \sin^3 t\,\mathbf{j} + t\,\mathbf{k}, \quad 0 \le t \le \pi/2.$$

Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

4.(14 pts.) Use the Green's Theorem to evaluate the line integral of

$$\mathbf{F}(x, y) = \left(e^x + \tan^{-1} y\right)\mathbf{i} + \left(3xy + \frac{x}{1+y^2}\right)\mathbf{j}$$

along the polar curve $r = 1 + \cos\theta$, $(0 \le \theta \le \pi)$.

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5.(14 pts.) Evaluate the line integral

$$\int_C (x+y^2)\,dx+(xz)\,dy+(y+z)\,dz$$

where C consists of line segments from $(\pi, 0, 0)$ to $(0, 0, 0)$, from $(0, 0, 0)$ to $(0, \pi, 0)$ and from $(0, \pi, 0)$ to $(0, 0, \pi)$.

6.(14 pts.) Find the area of the part of the surface $2x-y^2-3z^2=0$ that lies inside the elliptic cylinder $y^2+9z^2=1$.

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7.(15 pts.) Let S be the surface obtained by revolving the curve $(x-2)^2+z^2=1,\ z\geq 0$, about the z-axis. Evaluate the flux of $\mathbf{F}(x, y, z)=y\,\mathbf{i}-x\,\mathbf{j}+z\,\mathbf{k}$ across S where S is oriented upward.

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8.(15 pts.) Consider a polar curve

$C_1 : r=2-2\cos\theta,\ \frac{\pi}{3}\leq\theta\leq\pi$, in the xy-plane.

Let S be the surface obtained by revolving the curve C_1 about the x-axis and let $\mathbf{F}(x,y,z)=(x+e^z y^2+z^2 y)\,\mathbf{i}+(x^2\sin y+xy^2\cosh y)\,\mathbf{j}+(xy-e^{xz}\cos z)\,\mathbf{k}$. Evaluate

$$\iint_S \mathrm{curl}\mathbf{F}\cdot d\mathbf{S}$$

where S is oriented in the direction of the positive x-axis.

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9.(15 pts.) Let $\mathbf{F}(x,y,z) = \frac{1}{(x^2+y^2+z^2)^{3/2}}\,(x\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k})$. Evaluate the flux $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where S is the part of the paraboloid $z=2-x^2-y^2$, $-1\le z\le 2$, oriented upward.

10.(20 pts.)

(a) 행렬 $A=\begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$에 대해

$\det((2A)^T)+\det(A^2)$를 구하시오.

(b) 크레머 공식 (Cramer's Rule)을 이용하여, 다음 연립방정식의 해 중 w의 값을 구하시오.

$$\begin{cases} x+3y+5z+2w = 1 \\ -y+3z+4w = -1 \\ 2x+y+9z+6w = 0 \\ 3x+2y+4z+8w = 1 \end{cases}$$