Calculus II [MATH 162]

Midterm Exam (Fall 2022)

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1.(15 pts.) Find the limit, if it exists, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,\,y)\to(0,\,0)} \frac{(\sin x+\sin y)^2}{x^2+y^2}$

(b) $\displaystyle\lim_{(x,\,y,\,z)\to(0,\,0,\,0)} \frac{x^3yz+xy^3z-xyz^3}{x^2+y^2+z^2}$

2.(15 pts.) Answer the following questions.

(a) Sketch the region of integration and evaluate the integral.

$$\int_0^1\int_{x/2}^{2x} e^{y^2}\,dy\,dx+\int_1^4\int_{x/2}^{2} e^{y^2}\,dy\,dx$$

(b) Sketch the region of integration and express the sum of double integrals as a single iterated integral with reversed order of integration.
(Do **NOT** evaluate the integral.)

$$\int_{-1}^0\int_0^{\frac{2}{3}(x+1)} f(x,\,y)\,dy\,dx+\int_0^2\int_0^{\frac{2}{3}(x+1)} f(x,\,y)\,dy\,dx$$
$$+\int_2^6\int_0^{\sqrt{6-x}} f(x,\,y)\,dy\,dx+\int_2^6\int_{\sqrt{6-x}}^{2} f(x,\,y)\,dy\,dx$$

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3.(15 pts.) Answer the following questions if $f(x,y) = g(x^2y,\ e^{xy+1}-3,\ 3xy^2-2)$ and $g(r,s,t) = r^2+2s^2-t^2$.

(a) Use the chain rule to find $f_x(1,-1)$ and $f_y(1,-1)$.

(b) Find the linearization of $f(x,y)$ at $(1,-1)$.

(c) Use the result of (b) to estimate $f(1.01,-1.02)$.

4.(15 pts.) Find the local maximum and minimum values and saddle point(s) of the function

$$f(x,y) = 6x^3 - 3x^2y + \frac{15}{2}x^2 + y^2 + 4y.$$

5.(15 pts.) Suppose that the differentiable functions f and g satisfy the followings:

(1) f has the maximum rate of change at the point $P(x_0, y_0, z_0)$ in the direction of the vector $\langle 2, -1, 3 \rangle$ with the rate $2\sqrt{7}$.
(2) The tangent plane to the surface $g(x, y, z) = \sqrt{2}$ at the point P is parallel to the plane $x+2y-z=1$.
(3) $D_{\mathbf{u}}\, g(x_0, y_0, z_0) = 4$ for $\mathbf{u} = \frac{1}{3}\langle 2, -1, 2 \rangle$.
(4) $f(x_0, y_0, z_0) = \frac{1}{2}$ and $g(x_0, y_0, z_0) = \sqrt{2}$.

(a) Find the gradient vector ∇f at the point P.
(b) Find the gradient vector ∇g at the point P.
(c) Let $h(x, y, z) = f(x, y, z) g(x, y, z)$. Find the rate of change of h at the point P in the direction of the vector $\langle -1, 2, -2 \rangle$.

6.(15 pts.) Use Lagrange multipliers to find the absolute maximum and minimum values of the function $f(x, y) = x^3 + y^3 - \frac{3}{2}x^2y^2$ on the circle $x^2 + y^2 = 5$.

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7.(15 pts.) Answer the following questions.

(a) When the plane $x+y+mz=2$ is tangent to the ellipsoid $x^2+\frac{y^2}{2}+\frac{z^2}{2}=1$, find the positive real number m.

(b) For m in the result of (a), when the plane $x+y+mz=2$ is also tangent to the paraboloid $z=x^2+y^2+k$, find the real number k.

8.(15 pts.) Suppose that the lamina occupies the region

$$D=\{(x,y)\mid x^2+y^2\le 2x,\ x^2+y^2\ge 1,\ y\ge 0\}$$

with the density $\rho(x,y)=y$.

(a) Find the mass of the lamina.

(b) Find the x coordinate of the center of mass of the lamina.

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9.(15 pts.) Find the volume of the solid that lies within the cylinder $x^2+y^2=2y$, above the hemisphere $z=-\sqrt{4-x^2-y^2}$, and below the paraboloid $x^2+y^2=z$.

10.(15 pts.) Evaluate the integral.

$$\int_0^1\int_0^x\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}}\sqrt{x^2+y^2+z^2}\ dz\,dy\,dx + \int_1^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}}\sqrt{x^2+y^2+z^2}\ dz\,dy\,dx$$