Calculus II [MATH162]

Midterm Exam (Fall, 2022)

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1. (15pts) Find the limit, if it exists, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{4x^3+xy}{x^2+y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}\sin\left(\frac{1}{x^2+y^2}\right)$

2. (15pts) Let

$$f(x,y)=\begin{cases} \dfrac{x^3+3y^5}{x^2+x^2y^2+y^4} & ,\ (x,y)\neq(0,0) \\ 0 & ,\ (x,y)=(0,0) \end{cases}.$$

(a) Show that $f(x,y)$ is continuous at the point $(0,0)$.

(b) Find $f_x(0,0)$ and $f_{xx}(0,0)$.

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3. (15pts) Let $f(x,y,z)=x^2-2y^3+3z^4-2xyz$.
Suppose that there exists a differentiable function $g(x,y)$ such that $g(1,-1)=1$ and $f(x,y,g(x,y))=8$.
(a) Find the directional derivative of $g(x,y)$ at $(1,-1)$ in direction of the fastest increasing.
(b) Use the linear approximation to the estimate $g(1.3,\,-1.2)$.

4. (15pts) Let $g(s,t)=f\left(\frac{t}{s},\, s^2+t^2\right)$ and f be a differentiable function of x and y with $f_x(0,1)=2$, $f_y(0,1)=1$, $f_{xx}(0,1)=-1$, $f_{xy}(0,1)=3$ and $f_{yy}(0,1)=-2$.
(a) Find $g_s(1,0)$ and $g_{ss}(1,0)$.
(b) Find the function $h(x,y)$ satisfies

$$s\frac{\partial g}{\partial s}+t\frac{\partial g}{\partial t}=h(x,y)\frac{\partial f}{\partial y}$$

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5. (15pts) Let $f(x,y) = x^2 e^{-x} + y^2 - 2y$.

(a) Find and classify the critical points of the function $f(x,y)$.

(b) Use the Lagrange multipliers to find the extreme values of the function $f(x,y)$ on the region $D = \{(x,y) | x^2 + (y-1)^2 \le 1\}$.

6. (15pts) Find the volume of the solid that lies the surface $z = \cos y \tan^{-1}(\cos^2 y)$ and above the region $D = \left\{(x,y) \middle| \sin^{-1} x \le y \le \frac{\pi}{2}, 0 \le x \le 1\right\}$.

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7. (15pts)

(a) Use the polar coordinates to combine the sum

$$\int_{1/2}^{1}\int_{-\sqrt{3}x}^{-\sqrt{1-x^2}} f(x,y)\,dydx + \int_{1/2}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{3}x} f(x,y)\,dydx + \int_{1}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\ dydx$$

into one double integral.

(b) Find the center of mass above region if the density function is $\rho(x,y)=\sqrt{x^2+y^2}$.

8. (15pts)

(a) Rewrite the integral as an equivalent integral in the $dz\,dy\,dx$ order.

$$\int_0^1\int_0^{1-\sqrt{1-(y-1)^2}}\int_{-\sqrt{1-z}}^{\sqrt{1-z}} f(x,y,z)\,dx\,dz\,dy$$

(b) Convert the integral to an equivalent integral in spherical coordinates.

$$\int_0^{\pi}\int_0^{2\sin\theta}\int_{\sqrt{2r\sin\theta-r^2}}^{\sqrt{4-r^2}} f(r\cos\theta, r\sin\theta, z)\,rdz\,drd\theta$$

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9. (15pts) Evaluate $\iiint_E (1+x+y)\,dV$, where E is the solid that lies within the hyperboloid one-sheet $x^2+y^2-z^2=1$, above the plane $z=0$, and below the paraboloid $z=3-x^2-y^2$.

10. (15pts) Find the volume of the solid that lies above the cone $z=\sqrt{\frac{x^2+y^2}{3}}$ and below the surface $x^2+y^2+z^2-\sqrt{x^2+y^2+z^2}-z=0$.