Calculus I [MATH 161 (01~04)]　　Final exam　(Fall 2022)

Department :　　　　Id number :　　　　Name :

<u>단답형: (1번~5번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.</u>

1.(6 pts.) Find the first 3 nonzero terms of the Maclaurin series for $f(x)=\left(\tan^{-1}x\right)^2$.

2.(6 pts.) Find the minimum value of $A+B$ that satisfies

$$1-2\ln 2+\frac{4(\ln 2)^2}{2!}-\frac{8(\ln 2)^3}{3!}+\cdots=\frac{A}{B}$$

where A and B are natural numbers.(자연수)

3.(6 pts.) If the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is $\frac{\pi}{4}$ and $\boldsymbol{a}\times\boldsymbol{b}=\langle\sqrt{5},-2,4\rangle$, find $|\operatorname{proj}_{\mathrm{a}}\mathrm{b}\times\operatorname{proj}_{\mathrm{b}}\mathrm{a}|$.

4.(6 pts.) Let L be the line of intersection of the planes $3x-y-2z=1$ and $2x+y-2z=-1$.

(a) Find parametric equations for the line L.

(b) Find the distance from the point $(2,0,2)$ to L.

5.(6 pts.) Find parametric equations for the tangent line to the space curve obtained by taking intersection of the cylinder $x^2+4y^2=9$ and the plane $x-y+z=0$ at the point $(3,0,-3)$.

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단답형: (6번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

6.(6 pts.) If z is defined implicitly as a function of x and y by the equation $e^{xyz} = \ln\left[x^2 z(y+1)\right]$, find $\dfrac{\partial z}{\partial x}$ when $(x, y) = (1, 0)$.

7.(6 pts.) Find the directional derivative of the function $f(x, y, z) = \sin(xy) + yz$ at the point $\left(0, -\dfrac{\pi}{3}, \dfrac{\pi}{3}\right)$ in the direction of the vector $v = 3\mathbf{j} + 4\mathbf{k}$.

8.(6 pts.) Find an equation of the tangent plane of the surface $x^y + y^x + \tan^{-1}(x^2 + y^3 + z) = 2$ at the point $(1, 1, -2)$.

9.(6 pts.) The functions $a(x)$, $b(x)$ and $c(x)$ satisfy

$$\int_0^2 \int_{y^2}^{y+2} f(x)\, dx\, dy = \int_0^2 \int_0^{\boxed{a(x)}} f(x) dy dx + \int_2^4 \int_{\boxed{b(x)}}^{\boxed{c(x)}} f(x) dy dx.$$

Find $g(9)$, if $g(x) = a(x) - b(x) + [c(x)]^2$.

10.(6 pts.) Find the volume of the solid in the first octant that is bounded by the cylinders $x^2 + z^2 = 4$, $3z - x^2 = 0$ and the planes $x = 0$, $y = 0$ and $y = z$.

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<u>**서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.**</u>

11.(15 pts.) Let $f(x, y) = \sum_{n=0}^{\infty} \frac{y^{2n+2}}{(2n+1)!} x^n$.

(a) Find the value of $f\left(-1, \frac{\pi}{6}\right)$.

(b) Find the value of $g^{(4)}(0)$ if $g(x) = f_{yx}(x, -1)$.

12.(15 pts.) Let L_1 and L_2 be the lines

$$L_1 : x = 2t, \quad y = 0, \quad z = -t$$

$$L_2 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}.$$

(a) Find the value of $\cos\theta$ $(0 \le \theta \le \pi)$ where θ is the angle between the directional vectors of L_1 and L_2.

(b) Show that the lines L_1 and L_2 are skew.

(c) Find the distance between L_1 and L_2.

13.(15 pts.) Let P be a parallelopiped whose adjacent edges a, b, c are parallel to the vectors $\boldsymbol{v}_a = < -1, 2, 2 >$, $\boldsymbol{v}_b = < 1, 0, -1 >$ and $\boldsymbol{v}_c = < 0, -1, 1 >$ with lengths x, y and z, respectively.

(a) Find the volume of P as a function of x, y and z.

(b) Find the maximum value of $f(x, y, z)$ on $x + y + z = 12$.

14.(15 pts.) Find the maximum and minimum values of $f(x, y, z) = xyz$ on $x^2 + y^2 + z^2 \leq 1$.

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15.(15 pts.) Let $f(x,y) = x^3 + 3xy^2 - 15x + y^3 - 15y$.

(a) Find all critical points of $f(x,y)$.

(b) Find the local maximum and minimum values, and saddle point(s) of $f(x,y)$.

16.(15 pts.) Evaluate the following double integrals.

(a) $\displaystyle\int_{-1}^{1}\int_{|y|}^{1} e^{1+x^2}\,dx\,dy.$

(b) $\displaystyle\int_{0}^{2}\int_{\sin^{-1}\left(\frac{y}{2}\right)}^{\frac{\pi}{2}} \frac{1}{2+2\cos^2 x}\,dx\,dy.$