Calculus I [MATH 161 (22~24)] Final Exam (Spring 2023)

Department : Id number : Name :

단답형: (1번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

1. (6 pts.) Let $\boldsymbol{a} = \langle 1, 2, -3 \rangle$ and $\boldsymbol{b} = \langle 2, 0, 1 \rangle$.
Find $\boldsymbol{a} \cdot \boldsymbol{b}$ and $\boldsymbol{a} \times \boldsymbol{b}$.

2. (6 pts.) Find the distance from the point $(1, -2, 4)$ to the plane $\frac{3}{2}x + y + 3z = \frac{5}{2}$.

※ **(3~4)** Let $\boldsymbol{\alpha} = \langle 0, 1, a \rangle$, $\boldsymbol{\beta} = \langle 1, 0, 2 \rangle$, and $\gamma = \langle 3, -2, b \rangle$. Suppose that

ⅰ) $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and γ lie in the same plane and

ⅱ) $\operatorname{proj}_{\boldsymbol{\beta}} \boldsymbol{\alpha} = \operatorname{proj}_{\boldsymbol{\beta}} \gamma$.

3. (6 pts.) Find the real numbers a and b.

4. (6 pts.) Find the area of the parallelogram determined by the vectors $\boldsymbol{\beta}$ and γ.

5. (6 pts.) Find the tangent plane to the function $f(x, y) = 4\tan^{-1}\left(\frac{x}{y}\right)$ at $(1, 1)$.

1	$\boldsymbol{a} \cdot \boldsymbol{b} =$	$\boldsymbol{a} \times \boldsymbol{b} =$
2		
3	$a =$	$b =$
4		
5		

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6. (6 pts.) Find $f_{xy}(1,\ -1)$ and $f_{yy}(1,\ -1)$ of the function $f(x,\ y)=\dfrac{y}{2x+3y}$.

7. (6 pts.) Find the number of all critical points of the function $f(x,\ y)=\ e^{-(x^2+y^2)}\,(4xy)$.

8. (6 pts.) If $z=\tan^{-1}(x^2+y^2)$, where $x=s\ln t$ and $y=te^s$, then find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $s=0$ and $t=e$.

9. (6 pts.) Find the volume of the solid enclosed by the parabolic cylinder $y=4-x^2$ and the planes $y=0$, $z=0$, and $2y+z=8$.

10. (6 pts.) Find the volume of the solid that lies inside the sphere $x^2+y^2+z^2=3$ and above the paraboloid $z=x^2+y^2-1$.

6	$f_{xy}(1,\ -1)=$	$f_{yy}(1,\ -1)=$
7		
8	$\dfrac{\partial z}{\partial s}=$	$\dfrac{\partial z}{\partial t}=$
9		
10		

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서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.

11. (10 pts.) Find the distance between the given skew lines

$$L_1 : x = t, \quad y = 3, \quad z = 2t\,;$$
$$L_2 : 3 - x = \frac{y-4}{4} = \frac{z-1}{2}.$$

12. Answer the following questions:

(1) (5 pts.) Find the directional derivative $D_{\boldsymbol{u}} f(x,y)$ of the function $f(x, y, z) = y^{xz}$ at the point $(2, e, 1)$ in the direction of the vector $\boldsymbol{u} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$.

(2) (10 pts.) Suppose that the maximum rate of change of a differentiable function $w = f(x, y, z)$ at the point $P(1, 2, 3)$ is 6 and the direction in which it occurs is $\langle 2, 2, -1 \rangle$. Find $\nabla f(P)$ and the directional derivative of f at the point P in the direction of $\langle 1, -1, -1 \rangle$.

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13. Let P_1 be the tangent plane to the surface $xyz = -2$ at the point $Q(1,\,2,\,-1)$, and P_2 be the tangent plane to the surface $3x^2 + y^2 + 5z^2 = 12$ at the point Q. Answer the following questions:

(1) (9 pts.) Find symmetric equations for the line of intersection between the planes P_1 and P_2.

(2) (6 pts.) Find the distance from the point $R(0,\,2,\,4)$ to the line in (1).

14. Let $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2$. Answer the following questions:

(1) (7 pts.) Classify the critical points of $f(x,y)$.

(2) (8 pts.) Find the Maximum and minimum values of $f(x,y)$ on the region

$$D = \{\,(x,y) \mid x^2 + y^2 \le 16,\ x \ge 0,\ y \ge 0\,\}.$$

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15. Evaluate the double integrals:

(1) (10 pts.) $\int_0^1 \int_{\sin^{-1} y}^{\pi/2} \sec^2(\cos x)\, dx\, dy$

(2) (10 pts.) $\int_1^2 \int_0^{\sqrt{4-x^2}} 48\,(x^2+y^2)^{-2}\, dy\, dx$

16. (15 pts.) Find all numbers $a, b,$ and c which minimize the double integral

$$\int_0^1 \int_0^{1-y^2} \frac{2\,x\,y\,(4a^2x^2+3b^2x+2c^2)}{1-x}\, dx\, dy\,,$$

where $a, b,$ and c satisfy the relation $a^2+2b+c=5$.