Calculus I [MATH 161 (22~24)]

Department : Id number : Name : 단답형: (1번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것. 1. (6 pts.) Let $\boldsymbol{a} = \langle 1, 2, -3 \rangle$ and $\boldsymbol{b} = \langle 2, 0, 1 \rangle$. Find $\boldsymbol{a} \cdot \boldsymbol{b}$ and $\boldsymbol{a} \times \boldsymbol{b}$. 2. (6 pts.) Find the distance from the point (1, -2, 4)to the plane $\frac{3}{2}x + y + 3z = \frac{5}{2}$. **** (3~4)** Let $\boldsymbol{\alpha} = \langle 0, 1, a \rangle$, $\boldsymbol{\beta} = \langle 1, 0, 2 \rangle$, and $\gamma = \langle 3, -2, b \rangle$. Suppose that i) $\pmb{lpha},\ \pmb{eta},\ \text{and}\ \pmb{\gamma}$ lie in the same plane and ii) $\operatorname{proj}_{\beta} \alpha = \operatorname{proj}_{\beta} \gamma$. **3.** (6 pts.) Find the real numbers a and b. 4. (6 pts.) Find the area of the parallelogram determined by the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

5. (6 pts.) Find the tangent plane to the function $f(x, y) = 4 \tan^{-1} \left(\frac{x}{y}\right)$ at (1, 1).

1	$a \cdot b =$	a imes b =
2		
3	a=	<i>b</i> =
4		
5		

Final Exam (Spring 2023)

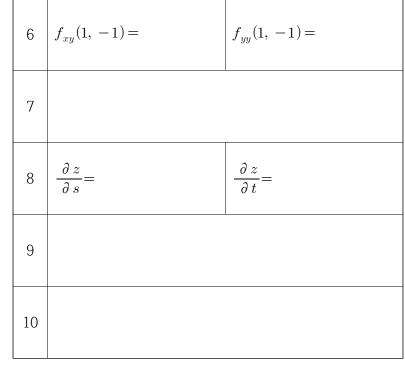
6. (6 pts.) Find $f_{xy}(1, -1)$ and $f_{yy}(1, -1)$ of the function $f(x, y) = \frac{y}{2x+3y}$.

7. (6 pts.) Find the number of all critical points of the function $f(x, y) = e^{-(x^2 + y^2)} (4xy)$.

8. (6 pts.) If $z = \tan^{-1}(x^2 + y^2)$, where $x = s \ln t$ and $y = te^s$, then find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when s = 0 and t = e.

9. (6 pts.) Find the volume of the solid enclosed by the parabolic cylinder $y = 4 - x^2$ and the planes y = 0, z = 0, and 2y + z = 8.

10. (6 pts.) Find the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 3$ and above the paraboloid $z = x^2 + y^2 - 1$.



서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.

11. (10 pts.) Find the distance between the given skew lines

$$\begin{array}{ll} L_1: x=t, & y=3, & z=2t \ ; \\ L_2: 3-x=\frac{y-4}{4}=\frac{z-1}{2} \ . \end{array}$$

12. Answer the following questions:

(1) (5 pts.) Find the directional derivative $D_{u} f(x,y)$ of the function $f(x, y, z) = y^{xz}$ at the point (2, e, 1)in the direction of the vector $u = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$.

(2) (10 pts.) Suppose that the maximum rate of change of a differentiable function w = f(x, y, z) at the point P(1, 2, 3) is 6 and the direction in which it occurs is $\langle 2, 2, -1 \rangle$. Find $\nabla f(P)$ and the directional derivative of f at the point P in the direction of $\langle 1, -1, -1 \rangle$.

13. Let P_1 be the tangent plane to the surface

xyz = -2 at the point Q(1, 2, -1), and P_2 be the

tangent plane to the surface $3x^2 + y^2 + 5z^2 = 12$ at the point Q. Answer the following questions:

- (1) (9 pts.) Find symmetric equations for the line of intersection between the planes P_1 and P_2 .
- (2) (6 pts.) Find the distance from the point R(0, 2, 4) to the line in (1).

14. Let $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$. Answer the following questions:

- (1) (7 pts.) Classify the critical points of f(x, y).
- (2) (8 pts.) Find the Maximum and minimum values of f(x, y) on the region

$$D = \left\{ (x, y) \mid x^2 + y^2 \le 16, \ x \ge 0, \ y \ge 0 \right\}.$$

15. Evaluate the double integrals:

- (1) (10 pts.) $\int_0^1 \int_{\sin^{-1}y}^{\pi/2} \sec^2(\cos x) \, dx \, dy$ (2) (10 pts.) $\int_1^2 \int_0^{\sqrt{4-x^2}} 48 \, (x^2+y^2)^{-2} \, dy \, dx$

16. (15 pts.) Find all numbers a, b, and c which minimize the double integral

$$\int_0^1 \int_0^{1-y^2} \frac{2 x y \left(4 a^2 x^2 + 3 b^2 x + 2 c^2\right)}{1-x} \, dx \, dy \,,$$

where a, b, and c satisfy the relation $a^2 + 2b + c = 5$.