Calculus I [MATH 161 (22~24)]   **Midterm Exam (Spring 2023)**

Department :   Id number :   Name :

단답형: (1번~5번) 단답형의 답은 페이지 하단에 주어진
네모 칸에 써야 점수 인정받습니다. 주의할 것.

1. (6 pts.) Find the exact value of $\sec\left(\tan^{-1}(-2)\right)$.

※ (2-3) Consider the curve $xy + \ln(x+y) = 0$.

2. (6 pts.) Find the value of $\dfrac{dy}{dx}$ at the point where $x = 0$.

3. (6 pts.) Find the value of $\dfrac{d^2y}{dx^2}$ at the point where $x = 0$.

4. (6 pts.) If $\sinh A = \dfrac{4}{3}$, then find the value of $\cosh A + \tanh A$.

5. (6 pts.) Evaluate the integral $\displaystyle\int_0^{\sqrt{5}} \dfrac{x^3}{\sqrt{4+x^2}}\,dx$.

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단답형: (6번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

6. (6 pts.) Evaluate the integral $\displaystyle\int_0^\infty \frac{\tan^{-1}x}{x^2+1}dx$.

7. (6 pts.) Find the number of intersections of the curves $r=1+2\cos 2\theta$ and $r=1$.

8. (6 pts.) Three polar curves $r=2\cos\theta$, $\theta=\dfrac{\pi}{6}$, and $r=\dfrac{1}{\cos\theta}$ partition the plane into several region. Find the area of the smallest region.

9. (6 pts.) Find the radius of convergence of the power series $\displaystyle\sum_{n=2}^\infty \frac{2^n(x-1)^n}{5n\ln n}$.

10. (6 pts.) Find the sum of the series $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n}{3^n(2n+1)}$.

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서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.

11. (15 pts.) Answer the following questions.

(1) Find the limit $\displaystyle \lim_{x \to 1} \frac{\tan^{-1} x - \sin^{-1}\left(\dfrac{x}{\sqrt{2}}\right)}{x^x - 1}$.

(2) Evaluate the integral $\displaystyle \int_{-1}^{2} \frac{1}{\sqrt{|x|}}\, dx$.

12. (15 pts.) Evaluate the following integrals.

(1) $\displaystyle \int_0^2 6x\sqrt{3 + 2x - x^2}\, dx$

(2) $\displaystyle \int_0^1 \frac{x^3 + 3x^2 + 9x + 3}{(x+1)^2(x^2+1)}\, dx$

13. (15 pts.) Consider the two polar curves
$$r = 1 + 2\sin\theta \quad \text{and} \quad r = 2\sin\theta.$$
(1) Find the area of the region that lies inside the larger loop of the curve $r = 1 + 2\sin\theta$.

(2) Find the intersection points of the above two curves.

(3) Find the area of the region lies inside $r = 2\sin\theta$ and outside inner loop of the curve $r = 1 + 2\sin\theta$.

14. (15 pts.) Determine whether the following series are convergent or divergent.

(1) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

(2) $\displaystyle\sum_{n=3}^{\infty} \frac{2}{n\ln n(\ln(\ln n))^2}$

(3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \tan^{-1}\left(\frac{1}{\sqrt{n}}\right)$

15. (15 pts.) Consider the power series $\displaystyle\sum_{n=0}^{\infty}\frac{1}{\cosh n}x^n$.

(1) Find the radius of convergence of the series.

(2) Find the interval of convergence of the series.

16. (15 pts.) Consider the function $\displaystyle f(x)=\int_0^{2x}\frac{\sin t}{t+2}dt$

for $|x|<1$. Answer the following questions.

(1) Find the Maclaurin series for $\dfrac{1}{x+1}$.

(2) Find the Maclaurin series for $\sin(2x)$.

(3) Let $\displaystyle\sum_{n=0}^{\infty}a_n x^n$ be the power series for $f(x)$. Find

the coefficients a_0, a_4, and a_5.