Calculus I (MATH 161)

Final Exam (Spring, 2023)

Department : Id number : Name :

풀이 과정을 자세히 기술해야 합니다.

1. (15pts) Test the series for convergence or divergence. Give reasons for your answers.

(a) $\sum_{n=1}^{\infty} n e^{-\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{\coth n}{n^2}$

(c) $\sum_{n=1}^{\infty} (-1)^n \left(\sec \frac{1}{n^3} - 1\right)$

2. (10pts) Find all positive values of p for which the series is convergent.

$$\sum_{n=1}^{\infty} (\ln n)^2 \sin\left(\frac{1}{n^p}\right)$$

Department : Id number : Name :

3. (10pts)

Suppose that $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent.

(a) Determine whether the series is convergent or divergent. Give a reason for your answer.

$$\sum_{n=1}^{\infty} \sqrt{1-\cos(a_n)}$$

(b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} (1+a_n)x^n$.

4. (10pts) Show that

$$\sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{4n}{(2n)!}+\frac{(-1)^n}{5^n}\right)=\frac{2}{e}+\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Department : Id number : Name :

5. (10pts)

(a) Use the binomial series, find the Maclaurin series for $f(x) = \dfrac{x^2}{\sqrt{1+x^2}}$.

(b) Evaluate $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt{1+x^2}}\, dx$ correct to within $\dfrac{1}{10^3}$.

6. (10pts)

(a) Find $(\mathrm{proj}_{2\mathbf{a}} 3\mathbf{b}) \cdot (\mathrm{proj}_{4\mathbf{b}} \mathbf{a})$ if

$$\mathbf{a} \cdot \mathbf{b} = \frac{1}{2} \text{ and } |\mathbf{a} \times \mathbf{b}| = \frac{3}{2}.$$

(b) Suppose that

$\mathbf{u} = \langle -1, a, b \rangle$, $\mathbf{v} = \langle 1, 0, -1 \rangle$, and $\mathbf{w} = \langle -3, 2, 1 \rangle$

Determine a and b if $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 0$ and $\mathbf{u} - \mathbf{w}$ is orthogonal to $\mathrm{proj}_{\mathbf{v}} \mathbf{w}$.

Department : Id number : Name :

7. (15pts)

(a) Let P_1 be the plane that passes through the point $(3,-2,3)$ and is perpendicular to the plane $x+y-z=2$ and $3x-y+2z=1$.

And let P_2 be the plane that passes through the point $(4,-1,1)$ and contains the line

$$x=1-t, y=3t, z=3+2t$$

Find the parametric equation for L_1 if L_1 be the line of intersection of the planes P_1 and P_2.

(b) Find the symmetric equation for L_2 if L_2 is the line through the point $(1,-1,2)$ and perpendicular to both $2\mathbf{i}+\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}+2\mathbf{k}$.

(c) Calculate the distance between L_1 and L_2.

8. (10pts) Let C be the curve parameterized by

$$\mathbf{r}(t)=\langle R\sin(t)\cos(t), R\sin^2(t), R\cos(t)\rangle,\ 0\le t\le 2\pi$$

where R is a positive constant. Find the minimum of the curvature of C.

Department : Id number : Name :

9. (10pts) Let C be a curve parametrized by

$$\mathrm{r}(t) = \langle \sin(2t), -\cos(2t), 4t^2 \rangle$$

(a) Find T, N, and B at $(0, -1, 0)$.

(b) Find the osculating plane and normal plane at $(0, -1, 0)$.