27^{th} Internet Seminar "Harmonic Analysis Techniques for Elliptic Operators"

Description of the Course

The study of the Laplacian on $\mathbb{R}^n$ through the Fourier transform lies at the center of classical harmonic analysis. It is Plancherel's theorem that intimately links the space $\mathrm{L}^2(\mathbb{R}^n)$ with the theory of weak derivatives and a symbolic calculus for the Laplacian. Examples are

- the Littlewood–Paley (in)equality
$$\|u\|_2^2 = C \int_0^\infty \|f(-t\Delta)u\|_2^2 \, \frac{\mathrm{d}t}{t}$$
- the Riesz transform "estimates"
$$\|(-\Delta)^{\frac{1}{2}} u\|_2 = \|\nabla u\|_2$$
- or the fact that the resolvent $(\lambda - \Delta)^{-1}$ is given by a nice kernel that yields bounds in $\mathrm{L}^p(\mathbb{R}^n)$ for $p \neq 2$.

Over the last decades, the quest to generalize these properties to elliptic operators $Lu = -\mathrm{div}(A\nabla u)$ with bounded measurable coefficients has triggered the development of new techniques that led to a surge of spectacular results in elliptic and parabolic PDE-theory. We will give an introductory course that covers the cornerstones of this "L-adapted Fourier analysis". The generalization of the Riesz transform estimates is the famous Kato square root problem whose solution will also be presented in the lectures. A variety of very recent results relying on these techniques will be covered in the project phase.
We expect the participants to have a basic knowledge in functional analysis, bounded operators, foundations of Hilbert spaces and some familiarity with the Fourier transform and functions in one complex variable.

Structure of the Internet Seminar

The annual Internet Seminars introduce master, Ph.D. and postdoc students to varying subjects related to evolution equations. The course consists of three phases.

- **Phase 1 (October-February):** A weekly lecture will be provided via the ISem website as lecture notes and a video recording. These lectures will be self-contained, and references for additional reading will be provided. The weekly lecture will be accompanied by exercises, and the participants are supposed to solve these problems.
- **Phase 2 (March-June):** The participants will form small international groups to work on diverse projects which supplement the theory of Phase 1 and provide some applications.
- **Phase 3 (June 17 to June 21, 2024):** The final workshop takes place at the CIRM in Luminy (Marseille, France). There the project teams of Phase 2 will present their projects and additional lectures will be delivered by leading experts in the field.

The ISem team of 2023/24 consists of

- Moritz Egert (Darmstadt)
- Robert Haller (Darmstadt)
- Sylvie Monniaux (Marseille)
- Patrick Tolksdorf (Karlsruhe)

The website of the 27th ISem is

`https://www.mathematik.tu-darmstadt.de/analysis/lehre_analysis/isem27/`

If you have any questions or remarks you can contact us using the e-mail address

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