Calculus I [MATH 161]

Midterm Exam (Spring 2023)

Department : Id number : Name :

(1번~9번) 풀이과정을 자세히 기술해야 합니다.

1.(10 pts.)

(a) Suppose that $s(y)$ is the arc length function of the curve $x = y^4 - y^3$ starting from the point $(0, 0)$, where $y \geq 0$.

Find the differential ds and evaluate ds for $y = \frac{3}{4}$ and $dy = 0.02$.

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 1)$ if $x = y^4 - y^3$.

2.(12 pts.) Evaluate the integrals.

(a) $\int \sin^{-1}(2x)\, dx$

(b) $\int \frac{x^2}{\sqrt{x^2 - x}} dx$

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3.(10 pts.) Suppose that $f(x)$ is a continuous **even function**. Find a function $f(x)$ and a real number k such that

$$\int_{-x}^{x}(\sinh t-1)f(t)\,dt+(k-1)=\int_{-k}^{x}t^2f(t)\,dt\ -x-\tan^{-1}x$$

for all $x>0$.

4.(10 pts.) Find the value(s) of the constant C for which the integral

$$\int_1^{\infty}\left(\frac{C}{x+1}-\frac{2\cos\left(2\cot^{-1}\sqrt{x}\right)}{3x-1}\right)dx$$

converges. Evaluate the integral for this value(s) of C.

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5.(10 pts.)

Let $f(x)=\tanh^{-1}(\cos x)-\operatorname{sech}^{-1}(\sin x)$. Find $f\left(\frac{\pi}{100}\right)$.

6.(12 pts.)

(a) Sketch the graphs $r=2+2\sin 3\theta$ and $r=2$.

(b) Find the area of the region that lies inside the curve $r=2+2\sin 3\theta$ and outside the circle $r=2$.

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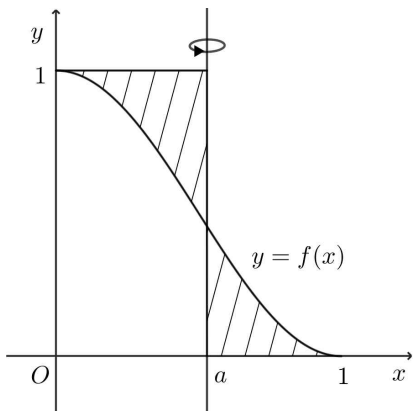
7.(12 pts.) Suppose that the continuous decreasing function $f(x)$ satisfies the followings:

(1) $f(0)=1, \quad f(1)=0$

(2) $\int_0^1 f(x)dx=\frac{1}{4}, \quad \int_0^1 xf(x)dx=\frac{1}{8}$

Let $V(a)$ be the volume of the solid obtained by rotating the region bounded by $y=f(x)$, $x=a$, $y=0$, and $y=1$ about the line $x=a$.
Find the minimum value of $V(a)$ on $0 \le a \le 1$.



8.(12 pts.)

Find $\lim_{x\to 0+} \frac{x\ln x}{(\sin(2x))^{\tan(2x)}-1}$.

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9.(12 pts.) Let C be the one arc of the cycloid given by

$$x = r(\theta - \sin\theta),\ y = r(1-\cos\theta),\ 0 \le \theta \le 2\pi \quad (r > 0)$$

Find the area of the surface obtained by rotating the curve C about the line $x = \pi r$.