Calculus II [MATH162] Final Exam (Fall, 2023)

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1. (10 pts.) Evaluate the integral by changing to spherical coordinates.

$$\int_0^{\pi/2}\int_{1/2}^{1}\int_{\sqrt{1-r^2}}^{\sqrt{3}r}(r^3+rz^2-r)\,dz\,dr\,d\theta + \int_0^{\pi/2}\int_1^2\int_0^{\sqrt{3}r}(r^3+rz^2-r)\,dz\,dr\,d\theta + \int_0^{\pi/2}\int_2^4\int_0^{\sqrt{4^2-r^2}}(r^3+rz^2-r)\,dz\,dr\,d\theta$$

2. (10 pts.) Let $\mathbf{F}(x,\,y,\,z)=\langle y^2z,\,2xyz+1,\,xy^2+x\rangle$ and $\mathbf{G}(x,\,y,\,z)=\langle 0,\,0,\,x\rangle$.

(a) Find the $\operatorname{curl}\mathbf{F}$, $\operatorname{curl}\mathbf{G}$ and $\operatorname{curl}(\mathbf{F}-\mathbf{G})$.

(b) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C consists of C_1 given by

$$\mathbf{r}_1(t)=\langle 3t,\,e^{t^2-t},\,t^2\rangle,\ \ 0\le t\le 1$$

followed by the line segment C_2 from $(3,\,1,\,1)$ to $(1,\,2,\,2)$.

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3. (10 pts.) Let C be the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$.

(a) Evaluate the line integral

$$\int_C (e^x+x^2\sinh^{-1}y)dx+(xy+\frac{x^3}{3\sqrt{1+y^2}})dy.$$

(b) Find the area of the region bounded by the curve C.

4. (15 pts.) Evaluate $\iint_S xy+1\,dS$, where S is the boundary of the region enclosed by the cylinder $x^2+y^2=1$ and the plane $z=0$ and the surface $z=\frac{1}{2}(x^2-y^2)+1$.

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5. (10 pts.) Find the flux of

$$\mathbf{F}(x,y,z) = \langle 2y\sin(x^2y),\, -3x\sin(x^2y),\, z \rangle$$

across the half-ellipsoid $\dfrac{x^2}{2}+\dfrac{y^2}{3}+z^2=1(z \geq 0)$ with upward orientation.

6. (10 pts.) Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ if

$$\mathbf{F}(x,y,z) = (e^{z^2}+z)\,\mathbf{i} + (x^2-2z^2)\,\mathbf{j} + (y^4+\sin(x^3))\,\mathbf{k}$$

and C is the curve with parametric equations

$$x = 2-2\sin t,\ y = 2\cos t,\ z = 2\sin t,\quad 0 \leq t \leq 2\pi.$$

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7. (15 pts.)

(a) 행렬A에 대해 다음 물음에 답하시오.

$$A=\begin{pmatrix} t^2(t-2) & 4-2t & 2-t \\ t & -2t & 1 \\ t & 2 & -t \end{pmatrix}$$

(i) $\det(A^{2023})=0$을 만족하는 t의 값을 모두 구하시오.

(ii) $t=3$일 때 $\det(A^{-1})$의 값을 구하시오.

(b) 크레머(Cramer's Rule)의 공식을 이용하여 다음 연립 방정식의 해 중 z의 값을 구하시오.

$$\begin{cases} 2x-4y+2z-w=3 \\ y-3z=-2 \\ x-4z=1 \\ y-z+2w=-4 \end{cases}$$

8. (15 pts.) Let $\mathbf{F}(x,y)=\langle x^2y-y,\ -xy^2\rangle$.

(a) Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ if C is given by

$$\mathbf{r}(t)=t\mathbf{i},\ -1\le t\le 1$$

(b) Find the simple piece-wise smooth curve C_1 from $(1,0)$ to $(-1,0)$ in the **upper half plane** $y\ge 0$ for which $\int_{C_1}\mathbf{F}\cdot d\mathbf{r}$ is a maximum.

(A curve is simple if it does not cross itself.)

(c) Find any simple piece-wise smooth curve C_2 from $(1,0)$ to $(-1,0)$ in the **whole xy-plane** for which $\int_{C_2}\mathbf{F}\cdot d\mathbf{r}$ is greater than $\int_{C_1}\mathbf{F}\cdot d\mathbf{r}$ in (b).

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9. (15 pts.)

Let $\mathbf{F}(x,y,z) = \dfrac{x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}}{(x^2+y^2+z^2)^{3/2}} + z^2\mathbf{k}$.

Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is given by

$$x^{2/3} + y^{2/3} + z^{2/3} = 1$$

oriented outward.