Calculus II [MATH162]

Midterm Exam (Fall, 2023)

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1. (15pts) Find the limit, if it exists, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y - xy^3}{(x^2+y^2)^2}$

(b) $\displaystyle\lim_{(x,y)\to(0+,0+)} \frac{x^2y(e^{-xy}-1)}{(x^2+y^2)^2}$

2. (10pts) Suppose that z is given implicitly as a function $z = g(x,y)$ by an equation of the form

$$e^{z-2}\cos(2x-z) = 2y^3 - x \text{ with } g(1,1) = 2.$$

Use differentials to estimate the value of $g(1.04, 0.99)$.

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3. (15pts)

Let $z=f(x,y)$ has continuous second-order partial derivatives where $x=e^{2s}-\sqrt{2}\cos t$, $y=e^{-s}+\sqrt{2}\sin t$. Use the table of values to calculate (a) and (b).

(x, y)	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
$(0, 2)$	1	-2	2	3	-1

(a) Find the second directional derivative of $f(x,y)$ at $(0,2)$ in the direction of the vector $\vec{v}=\langle 1,1\rangle$.

(b) Find the value of $\dfrac{\partial^2 z}{\partial s^2}$ when $s=0$ and $t=\dfrac{\pi}{4}$.

4. (10pts) Consider a rectangular box without a lid whose volume is $80\,cm^3$. The costs required to make the box are as follows:

The cost of the bottom of the box is 400won per cm^2.
The cost of the front of the box is 300won per cm^2.
The cost of the remaining sides of the box is 200won per cm^2.

Use Lagrange multipliers to find the width, length, and height of the box with the minimum cost.

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5. (10pts) Evaluate the double integral

$$\int_0^2 \int_{y/2}^{(4-y)/2} y \sin(x^3 - 1)\, dx\, dy + \int_1^2 \int_{4-2x}^{2x} y \sin(x^3 - 1)\, dy\, dx$$

6. (10pts) Find the center of mass of the lamina that occupies the region

$$D = \left\{(x,y) \mid 0 \le y \le \sin\beta,\ y\cot\beta \le x \le \sqrt{1-y^2}\right\}$$

where $0 < \beta < \pi/2$ if the density function is

$$\rho(x,y) = \sqrt{1-x^2-y^2}\,.$$

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7. (12pts) Find the volume of the region E bounded by $z=y^2$, $y+z=2$, $z=x$, $y=0$, and $x=2$.

8. (13pts) Evaluate the integral

$$\int_0^1 \int_{-\sqrt{(1-z^2)/2}}^{\sqrt{(1-z^2)/2}} \int_{\sqrt{y^2+z^2}}^{\sqrt{1-y^2}} z \, dx \, dy \, dz$$

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9. (15pts) A lamina is defined by the region

$$D=\{(x,y)\mid a\le x\le a+1,\ b\le y\le b+1\}$$

with the density function $\rho(x,y)=x^2(1+y^2)$. Here, let the mass of the lamina be $f(a,b)$.

(a) Find the local maximum, minimum values and saddle point(s) of the $f(a,b)$.

(b) Find the absolute maximum and minimum values of the $f(a,b)$ on the set $R=\{(a,b)\mid |a|\le 1 \text{ and } |b|\le 1\}$.