Calculus I [MATH 161 (01~04)]

Final Exam (Fall 2023)

Department : Id number : Name :

단답형: (1번~5번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

※ **(1-2)** Let $\mathbf{a}$ and $\mathbf{b}$ be 3-dimensional vectors such that $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \langle 1, -1, 1 \rangle$.

1. (6 pts.) Find the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$.

2. (6 pts.) Find $\text{Proj}_{\mathbf{a}}\mathbf{b} \times \text{Proj}_{\mathbf{b}}\mathbf{a}$.

3. (6 pts.) Find the distance from the point $A(2, 0, -1)$ to the line

$$l : x = t+1, \quad y = t, \quad z = t, \quad t \in \mathbb{R}.$$

4. (6 pts.) Find all constants a such that the vectors $\langle a, 1, -1 \rangle$, $\langle 2, 0, 1 \rangle$, and $\langle -2, 1, -3 \rangle$ are coplanar(동일 평면).

5. (6 pts.) Consider the surface $6yz + 3x^2 \ln y - z^2 = 0$. Find $\dfrac{\partial z}{\partial y}$ at the point $\left(0, \dfrac{e}{3}, 2e\right)$.

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단답형: (6번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

6. (6 pts.) Let $f(x,y)=\begin{cases}\dfrac{\sin(x^3+y^4)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0).\end{cases}$

Find $f_x(0,0)$.

7. (6 pts.) Use the Chain Rule to find the partial derivative $\dfrac{\partial w}{\partial r}$ of

$w=xy+yz+zx,\ x=r\cos\theta,\ y=r\sin\theta,\ z=r\theta$

when $r=1, \theta=0$.

8. (6 pts.) Let $f(x,y,z)=xy^2\tan^{-1}z$. Find the directional derivative of f at the point $(2,1,1)$ in the direction of the vector $\mathbf{v}=4\,\mathbf{i}+\mathbf{j}$.

9. (6 pts.) Find the positive number a such that the plane $2x+ay+z=9$ is tangent to the paraboloid $2x^2+y^2+3z=0$.

10. (6 pts.) Evaluate the double integral

$$\int_0^2\int_0^{\sqrt{4-x^2}}(x^2+y^2)\,dy\,dx.$$

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서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.

11. (15 pts.) Consider the tetrahedron with the vertices $A(3, 1, 4)$, $B(2, 0, 0)$, $C(-1, 2, 1)$, $D(0, -2, 2)$.

(1) Find parametric equations of each line L_1 and L_2 containing $\overline{AB}$ and $\overline{CD}$, respectively.

(2) Find the distance between the skew lines L_1 and L_2.

(3) Find the volume of the tetrahedron.

12. (15 pts.) Find parametric equations for the tangent line to the curve of intersection of the surface $z = \dfrac{1}{xy}$ and the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$.

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13. (15 pts.) Find the local maximum and minimum values and saddle points of the function

$$f(x,y)=2x^3+3x^2y-18x+y^3-9y+2.$$

14. (15 pts.) Find the minimum volume for a solid bounded by the planes $x=0$, $y=0$, $z=0$ and a tangent plane to the ellipsoid $x^2+\frac{y^2}{3}+\frac{z^2}{4}=1$ at a point (x_0, y_0, z_0) in the first octant.

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15. (15 pts.) Find the volume of the solid bounded by parabolic cylinders $z=2-x^2$, $z=x^2$, and the planes $3z+y-15=0$, $y=0$.

16. (15 pts.) Evaluate the following double integrals.

(1) $\displaystyle\int_0^1\int_{\sqrt{x}}^1 \frac{4y}{y^8+1}\,dy\,dx$

(2) $\displaystyle\int_0^{\pi/4}\int_0^{\sec\theta} r^2(\cos\theta+\sin\theta)\,dr\,d\theta + \int_{\pi/4}^{\pi/2}\int_0^{\csc\theta} r^2(\cos\theta+\sin\theta)\,dr\,d\theta$