

Department :

Id number :

Name :

단답형: (1번~5번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

1. (6 pts.) Evaluate the exact value of

$$\sin\left(2\tan^{-1}\left(\frac{1}{2}\right)\right) + \cos\left(2\tan^{-1}\left(\frac{1}{2}\right)\right).$$

2. (6 pts.) Find the exact value of

$\ln(\cosh(4) - \sinh(4))$. Simplify where possible.

3. (6 pts.) Consider the curve

$$x^3 - 2(x-1)y + \ln\left(\frac{2x-1}{y}\right) = 2.$$

Find the value of $\frac{dy}{dx}$ at the point (2, 3).

4. (6 pts.) Evaluate the integral

$$\int_1^2 \frac{2x^2 + x - 2}{x^2(x+1)} dx.$$

5. (6 pts.) Evaluate the integral

$$\int_{\sqrt{5}}^{2\sqrt{2}} \frac{6}{x\sqrt{9-x^2}} dx.$$

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단답형: (6번~10번) 단답형의 답은 페이지 하단에 주어진 네모 칸에 써야 점수 인정받습니다. 주의할 것.

6. (6 pts.) Evaluate the improper integral

$$\int_1^{\infty} \frac{\tan^{-1}x}{x^2} dx.$$

7. (6 pts.) For the polar curve $r=1+\sin 2\theta$, find the slope of the tangent line when $\theta=\pi/3$.

8. (6 pts.) Determine whether the series

$$\sum_{n=1}^{\infty} \left(\frac{4 \cdot 6 \cdot 8 \cdot \dots \cdot (2n+2)}{3 \cdot 6 \cdot 9 \cdot \dots \cdot (3n)} \right)^2$$

is convergent or divergent.

9. (6 pts.) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{9^n (2n)!}$.

10. (6 pts.) Let $\sum_{n=0}^{\infty} a_n x^n$ be the Maclaurin series for the function $f(x) = (\tan^{-1}x)^2$. Find the coefficient a_{11} of the Maclaurin series for $f(x)$.

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서술형: (11번~16번) 풀이 과정을 자세히 기술해야 합니다.

11. (15 pts.) Find the limit, if it exists. If the limit does not exist, explain why.

$$(1) \lim_{x \rightarrow 0} \frac{5^{x^2} - 3^{3x}}{\sin^{-1}(4x^2) - \tan^{-1}(2x)}$$

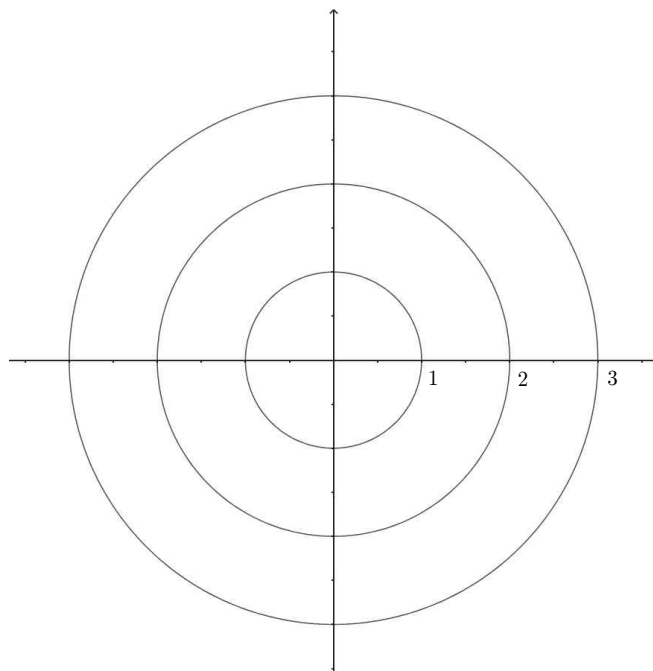
$$(2) \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x^2}\right)^{bx^2}, \quad (a, b > 0)$$

12. (15 pts.) Evaluate the integral

$$\int_1^{\infty} \frac{2x}{\sqrt{x^4 - 1}} dx.$$

13. (15 pts.) Answer the following questions.

- (1) Graph the polar curves $r = 2\sin\theta + 2\cos\theta$ and $r^2 = -4\sin 2\theta$.



- (2) Find the area of the region that lies inside both curves $r = 2\sin\theta + 2\cos\theta$ and $r^2 = -4\sin 2\theta$.

14. (15 pts.) Answer the following questions.

- (1) For which positive integers k the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

- (2) Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$ is convergent or divergent.

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15. (15 pts.) Consider the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x+2)^n}{n2^n}.$$

- (1) Find the radius of convergence of the series.
- (2) Find the interval of convergence of the series.

16. (15 pts.) Let $f(x) = x^2 \int_0^x \tanh^{-1} t \, dt$.

- (1) Find the Maclaurin series for the function $f(x)$.
- (2) Find $f^{(1004)}(0)$.