Calculus I (MATH 161) Final Exam (Spring, 2024)

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풀이 과정을 자세히 기술해야 합니다.

1. (10pts) Let $f(x) = \dfrac{x^3}{x^4 + 4}$.

(a) Find the Maclaurin series for $f(x)$.

(b) Use (a) to find $f^{(204)}(0)$.

2. (10pts)

(a) Find the area of the triangle with vertices

$$P(3, -2, 0),\ \ Q(3, 1, 3) \text{ and } R(4, 0, 2)$$

(b) Let L_1 be the line through the points P and Q, L_2 be the line through the points R and the origin. Calculate the distance between L_1 and L_2.

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3. (10pts) Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty}(3a_n - 2|a_n|)$, where $\sum_{n=1}^{\infty} a_n$ is conditionally convergent.

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{(n+2)^3}\,\ln(n+2)}$

c) $\sum_{n=2}^{\infty} \frac{\operatorname{sech} n}{(\ln n)^2}$

4. (10pts) Consider the power series $\sum_{n=3}^{\infty} \frac{(2x-1)^n}{5^n(\ln n)^3}$.

(a) Find the radius of convergence of the power series.

(b) For what values of x does the series converge?

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5. (10pts) Show that

$$\sum_{n=0}^{\infty}\frac{(2n)!}{16^n(n!)^2(2n+1)}=\frac{\pi}{3}$$

(hint : Use the Maclaurin series for $\sin^{-1}x$.)

6. (10pts) Suppose that two vectors $\mathbf{a}$ and $\mathbf{b}$ have lengths $\sqrt{2}$ and 2, respectively, and the angle between them is $\pi/3$. Find the volume of the tetrahedron determined by three edge vectors $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$, $\mathrm{proj}_{\mathbf{b}}\mathbf{a}$, and $\mathrm{proj}_{\mathbf{a}}\mathbf{b}\times\mathrm{proj}_{\mathbf{b}}\mathbf{a}$.

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7. (10pts) Let P_1 be the plane which contains the line $x=2+t$, $y=-2+5t$, $z=2t$ and is parallel to the vector $\langle 0, 3, -2\rangle$. Let P_2 be the plane which consists of all points equidistant from $A(2, -1, 0)$ and $B(4, -2, -3)$.

(a) Find the equations of planes P_1 and P_2.

(b) Find parametric equations of the line of intersection of the two planes.

8. (10pts) Let C be the curve of intersection of the parabolic cylinder $y^2=2x$ and the surface $3z=xy$.

(a) Find parametric equations for the curve C and the length of the curve C from the origin to the point $\left(2, 2, \frac{4}{3}\right)$.

(b) Find the maximum value of the curvature of curve C.

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9. (10pts) Consider the curve C defined by the vector equation

$$\mathbf{r}(t) = \langle 5\cos t,\ 3\sin t - 4t,\ 4\sin t + 3t \rangle.$$

(a) Reparametrize the curve C with respect to arc length measured from $(5, 0, 0)$ in the direction of increasing t.

(b) Find $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$, and the osculating plane at $(5, 0, 0)$.

10. (10pts) Let $\mathbf{a} = \langle 1, -2, 2 \rangle$, $\mathbf{b} = \langle 3, 4, 0 \rangle$.

(a) If $\mathbf{v}_1 = \mathbf{b} \times \mathbf{a}$, $\mathbf{v}_2 = (\mathbf{b} \times \mathbf{a}) \times \mathbf{a}$, $\mathbf{v}_3 = ((\mathbf{b} \times \mathbf{a}) \times \mathbf{a}) \times \mathbf{a}$,

$\mathbf{v}_4 = (((\mathbf{b} \times \mathbf{a}) \times \mathbf{a}) \times \mathbf{a}) \times \mathbf{a}$, $\cdots$,

then show that $\mathbf{v}_n$ is orthogonal to $\mathbf{a}$ for all n.

(b) Find the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ if a_n is the length of $\mathbf{v}_n$, by using (a).

(c) If $\mathbf{u}_1 = \mathbf{a} \times \mathbf{b}$, $\mathbf{u}_2 = (\mathbf{a} \times \mathbf{b}) \times \mathbf{a}$, $\mathbf{u}_3 = ((\mathbf{a} \times \mathbf{b}) \times \mathbf{a}) \times \mathbf{b}$,

$\mathbf{u}_4 = (((\mathbf{a} \times \mathbf{b}) \times \mathbf{a}) \times \mathbf{b}) \times \mathbf{a}$, $\mathbf{u}_5 = ((((\mathbf{a} \times \mathbf{b}) \times \mathbf{a}) \times \mathbf{b}) \times \mathbf{a}) \times \mathbf{b}$, $\ldots$,

then find the radius of convergence of $\sum_{n=1}^{\infty} b_n x^n$,

where b_n is the length of $\mathbf{u}_n$.

(you can use the formula $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ without proof)