# Calculus I [MATH 161]

## Midterm Exam (Spring 2024)

Department : Id number : Name :

(1번~10번) 풀이과정을 자세히 기술해야 합니다.

1.(10 pts.)

(a) Find the exact value of $\csc(\sin^{-1}(\pi/4))$.

(b) Evaluate the definite integral

$$\int_0^1 \frac{x^2}{(x^2+1)^{3/2}}\,dx$$

2.(10 pts.)

(a) If $f(x)=\int_0^{\tanh x}\sqrt{1-t^2}\,dt$ and $g(y)=\int_0^y f(x)dx$, find $g''(\ln 2)$.

(b) Find the exact length of the curve.

$$y=\cos^{-1}x-\sqrt{1-x^2},\quad 0\le x\le 1$$

3.(10 pts.) (a) For what value of the constant $k$ is the function $f$ continuous for all $x$ in $\mathbb{R}$?

$$f(x)=\begin{cases}\dfrac{1}{\sinh x}\displaystyle\int_1^{\cosh x}(1+e^{t^2})dt\,, & x\neq 0\\ k & , \ x=0\end{cases}$$

(b) Find the linearization of $f(x)$ at $a=0$.

4.(10 pts.) Assume that $0\le f'(x)\le f(x)$ for all $1<x<3$ and $f(2)=0$. Use the mean value theorem to show that $f(x)=0$ for all $1<x<3$.

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5.(10 pts.) Evaluate the limits.

(a) $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sinh^{-1}x}\right)$

(b) $\lim_{x\to 0^+}(\csc x)^{\frac{\cos^{-1}x}{\ln x}}$

6.(10 pts.) The integral represents the volume of the solid obtained by rotating the region $R$ about the $x=\frac{\pi}{2}$.

$$\int_0^1 \pi\left[\left(\frac{\pi}{2}-\sin^{-1}\left(\sqrt{y}\right)\right)^2-\left(\frac{\pi}{2}-\sin^{-1}\left(\sqrt[4]{y}\right)\right)^2\right]dy$$

Use the method of cylindrical shells to find the volume of the solid.

7.(10 pts.) Determine whether the integral is convergent or divergent. Evaluate the integral if it converges.

$$\int_1^{\infty} \frac{-5x^2+10x}{(2x+1)^2(x^2+1)}\,dx$$

8.(10 pts.) Find the area of the surface obtained by rotating the curve $x=\sqrt{4-y^2}$ about the line $x=2$.

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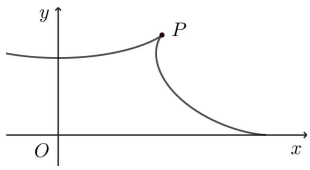
9.(10 pts.)

(a) Sketch the curves $r=\sqrt{2}\sin(2\theta)$ and $r^2=3\cos(2\theta)$.

(b) Find the area inside the polar curve $r=\sqrt{2}\sin(2\theta)$ and outside the polar curve $r^2=3\cos(2\theta)$.

10.(10 pts.) The curve $C$ is defined by the parametric equation

$$x=5\cos t+\cos(5t),\ y=5\sin t-\sin(5t),\ 0\le t\le\frac{2}{3}\pi.$$




(a) Find an equation of the tangent to the curve $C$ at $P(t=\frac{\pi}{3})$.

(b) Find the area enclosed by the curve $C$, the tangent to the curve $C$ at $P$ and $x$-axis.