Calculus II (MATH 162) Final Exam (Fall, 2024)

Department : Id number : Name :

<u>풀이 과정을 자세히 기술해야 합니다.</u>

1. (10pts)

(a) Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F}(x,y) = \langle y - \cos y,\ x\sin y + y \rangle$, where C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented <u>counterclockwise.</u>

(b) Find the area of the pentagon with vertices $(0,0)$, $(2,1)$, $(1,3)$, $(0,2)$, and $(-1,2)$.

2. (10pts) Let C be the curve given by

$$C : \mathbf{r}(t) = t^3\,\mathbf{i} - t^2\mathbf{j} + t\ \mathbf{k}\ ,\ 0 \le t \le 1$$

(a) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x,y,z) = (x+y^2)\,\mathbf{i} + xy\,\mathbf{j} + (y+z)\,\mathbf{k}$$

(b) Evaluate $\int_C \sin y\, dx + (x\cos y + \cos z)\,dy - y\sin z\, dz$.

Department : Id number : Name :

3. (10pts) Let $g(x,y,z)=\int_0^{x+y-2z} f(u)\,du$ with

$$\int_0^1 f(u)du=5 \text{ and } \int_0^1 uf(u)\,du=3$$

(a) Evaluate

$$\int_C f(x+y-2z)dx+f(x+y-2z)dy-2f(x+y-2z)dz,$$

where C is any smooth curve from $(1, 3, 2)$ to $(2, 5, 3)$.

(b) Evaluate $\int_C g(x, y, z)\,ds$, where C is the line segment from $(1, 3, 2)$ to $(2, 5, 3)$.

4. (10pts) Let $\mathbf{F} = \frac{x-y}{x^2+y^2}\,\mathbf{i}+\frac{x+y}{x^2+y^2}\,\mathbf{j}$ and C be the curve given by $\mathbf{r}(t)=t\,\mathbf{i}+(2t^2-6)\,\mathbf{j},\ -2 \le t \le 2$. Find the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

Department : Id number : Name :

5. (10pts) Let S be the surface given by

$$S : x^2+y^2-z^2=1$$

(1) Find a parametric representation for the surface S and the inward (toward z-axis) unit normal vector $\mathbf{n}$ of S.

(2) Find the area of the part of the surface S that lies between the plane $z=0$ and the plane $z=1$.

6. (10pts) Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ if

$$\mathbf{F} = \langle 3ze^{x^2-y^2}, 2ze^{x^2-y^2}, 2x-4y+e^{z^2} \rangle$$

and S is the part of the sphere $x^2+y^2+z^2=4$ that lies in the half-space $y \geq x$ with <u>outward</u> orientation of the sphere.

Department : Id number : Name :

7. (10pts) Find the flux of

$$\mathbf{F}(x,y,z) = \left\langle -z^2 e^{(xy)^2} + \tan^{-1}(xz),\ z^2 e^{(xy)^2} - \frac{yz}{1+x^2z^2},\ -z+1 \right\rangle$$

across the paraboloid $z = 4 - x^2 - y^2,\ z \geq 0$ with <u>downward</u> orientation.

8. (10pts)

(a) 연립방정식 $\begin{cases} x+y+z &= u \\ x-y+z &= v \\ x+y+2z &= w \end{cases}$ 에 대하여

크레머 룰(Cramer's rule)을 이용하여, y를 구하시오.

(b) 아래 식을 만족하는 (x,y,z)의 집합 E에 대하여

$$(x+y+z)^2 + (x-y+z)^2 + (x+y+2z)^2 \leq 1,\ x+y+2z \geq 0$$

위의 (a)를 이용하여, 다음 적분을 구하시오.

$$\iiint_E y^2\left((x+y+z)^2 + (x-y+z)^2 + (x+y+2z)^2\right) dV$$

Department : Id number : Name :

9. (10pts) 다음 행렬 A, B에 대하여 $\det((2A)^{-1}B^T)$를 구하시오.

$$A=\begin{pmatrix} 5 & 3 & -2 & 6 \\ 2 & 1 & -1 & 1 \\ 4 & 2 & 1 & 5 \\ 7 & 3 & 5 & 3 \end{pmatrix}, \quad B=\begin{pmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{pmatrix}$$

10. (10pts) Let $\mathbf{F}(x,y,z)=\dfrac{(z-y)\mathbf{i}+(x-z)\mathbf{j}+(y-x)\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$

and S is the <u>outward oriented</u> surface whose sides S_1 are given by cylinder $x^2+y^2=1$ with $0 \le z \le 3$, whose top S_2 is the part of the plane $z=3$ inside the cylinder. (S is a cylinder <u>without bottom.</u>)

(a) Use divergence theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

(b) Find a vector field $\mathbf{G}$ such that $\mathbf{F}=\nabla \times \mathbf{G}$.
(you don't need to explain how to find it.)

(c) Use (b) and Stokes' theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ again, as the line integral.