Calculus II [MATH 162]

Midterm Exam (Fall 2024)

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(1번~10번, 각 10점) 풀이 과정을 자세히 기술해야 합니다.

1. (a) Find and classify the critical points of the function $f(x, y) = x^4 + 4x^3 + 2y^2 - 8xy$.

(b) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ at $(1, 1, 1)$ if $xz + y e^{xy^2 - z} = 2z^2$.

2. Evaluate the integrals.

(a) $\int_0^1 \int_{\sqrt{y}}^1 \frac{1}{x^3+1} \, dx \, dy$

(b) $\iiint_E xy \, dV$, where E is enclosed by the surfaces $y = x^2$, $y = x$, $z = y$, and $z = 0$.

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3. Find the limit, if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x - x\cos y}{x^2 + y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x\sin y - y\sin x}{x^4 + y^4}$

4. Let a, b and c be positive real numbers such that $a^2 + b^2 + c^2 = 1$. Use the Lagrange multipliers to find the minimum volume for the tetrahedron bounded by the plane $ax + by + cz = 1$ and the three coordinate planes $x = 0$, $y = 0$, and $z = 0$.

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5. Suppose f is a differentiable function of u and v, and $g(x, y, z) = x^3 f\left(\frac{y}{x}, \frac{z}{x}\right)$.

(a) Find $x g_x + y g_y + z g_z - 3g$.

(b) If $f(1, 2) = 5$, use a linear approximation of $g(x, y, z)$ at $(2, 2, 4)$ to estimate $g(2.1, 2.1, 4.2)$.

6. Evaluate the integral $\iint_{D_1 \cup D_2} (x^2 + y^2)\, dA$, where D_1 and D_2 are the disks $x^2 + y^2 \le 2$ and $(x-1)^2 + y^2 \le 1$, respectively.

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7. Suppose that f has continuous second partial derivatives and $g(x, y) = \ln|f(x, y)|$.
If $f(a, b) = 3$, $D_{\boldsymbol{u}} f(a, b) = 3\sqrt{2}$, $D_{\boldsymbol{u}}^2 f(a, b) = 3$, and $\boldsymbol{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ for some real number a and b, find $D_{\boldsymbol{u}}^2 g(a, b)$.
($D_{\boldsymbol{u}}^2 f(a, b)$ is the second directional derivative of $f(x, y)$ at (a, b) in the direction of $\boldsymbol{u}$.)

8. Evaluate the integral by changing to spherical coordinates.

$$\left(\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^2 zr\,dz\,dr\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^{2\sqrt{3}}\int_{\frac{r}{\sqrt{3}}}^2 zr\ dz\,dr\,d\theta\right)$$
$$-\left(\int_0^{2\pi}\int_0^1\int_1^2 zr\ dz\,dr\,d\theta + \int_0^{2\pi}\int_1^2\int_r^2 zr\ dz\,dr\,d\theta\right)$$

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9. Find the volume of the solid that lies above the paraboloid $z=\frac{3}{4}(x^2+y^2)$ and inside the rectangular box $B=\{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$.

10. Let E be a solid that enclosed by the surface $(x^2+y^2+z^2)^2 = z(x^2+y^2)$. Evaluate $\iiint_E z\, dV$.